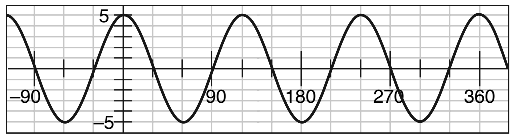
SKETCH AND INVESTIGATE

Q1



The parameter a controls the amplitude of the sinusoid.
The parameter b controls the period.

Q2 Answers will vary. The point of this question is to get students thinking about the process before they reveal the answer. Students have no basis yet for guessing correctly, so emphasize that the important thing is guessing, not getting the answer right.

Q3 The x-intercepts on the Cartesian graph correspond to $r = 0$, where the curve crosses the pole (origin) of the polar graph.

Q4 The maximum and minimum points on the Cartesian graph correspond to the outermost point of each leaf of the polar graph. The only difference between the two types of points on the polar graph is that the maximum points are created when the output value (r) is positive (the "bowtie" is on the positive side of the output bar), and the minimum points are created when the output value is negative (the "bowtie" is on the negative side of the output bar).

Q5 The polar graph starts on the positive side of the output axis for the first repetition (starting at 0°) and starts on the negative side of the output axis for the second repetition (starting at 180°). At all corresponding points (points whose θ-values are separated by 180°), r-values are opposite.

Q6 The parameter a controls the distance from the pole to the tip of each leaf.

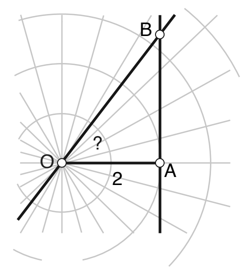
Q7 When the parameter b is odd, the number of leaves is equal to b. When b is even, the number of leaves is equal to $2b$.

EXPLORE MORE

Q8 Between 0° and 360° (not including 360°), the graph of $y = a\cos(bx)$ has b maximums and b minimums for a total of $2b$ extreme points—the points that become the outer points on the leaves. For even integer values of the parameter b, the Cartesian graph yields a maximum

y-value at $\theta = 0°$ and $\theta = 180°$, meaning that the graph doesn't double back on itself and there will be the maximum $(2b)$ number of leaves. For odd integer values of the parameter b, the Cartesian graph yields a maximum y-value at $\theta = 0°$, but it yields a minimum at $\theta = 180°$, causing the graph to double over itself at 180° and resulting in only b leaves.

Q9 This equation produces a vertical line 2 units to the right of the pole. Students can use trigonometry to explain why this occurs. Consider the right triangle in the following diagram, using an arbitrary point B on the vertical line through point A at $(2, 0°)$.



To find the length of the hypotenuse OB, use the cosine function:

$$\cos(\theta) = \frac{OA}{OB} = \frac{2}{OB}$$

$$OB = \frac{2}{\cos(\theta)} = 2\sec(\theta)$$

Thus the equation of any point on the vertical line must be $r = 2\sec(\theta)$.

Q10 Three interesting functions to try are

$$f(\theta) = a\tan(b\theta),$$

$$f(\theta) = a\left(\frac{\theta}{90°}\right),$$

$$\text{and } f(\theta) = a\sqrt{\frac{\theta}{90°}}$$